

# Direct Adaptive Neural Network Control for A Class of Ship Course Uncertain Discrete-time Nonlinear Systems

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## Abstract

In this paper, a direct adaptive radial basis function (RBF) neural network control algorithm is presented for a class of ship course with uncertain discrete-time nonlinear systems. To avoid some system states that are unmeasurable and make the adaptive control approach more universal and convenient to be implemented in practical application, the original ship course with uncertain discrete-time nonlinear system is transformed into the form of the input-output model. According to the input-output model, a direct adaptive RBF NN control for the ship course with discrete-time nonlinear system is carried out based on the existence of the implicit desired feedback control (IDFC). In the controller design process, RBF neural networks are used to emulate the desired feedback control and approximate the unknown function. The stability of the closed-loop system is proven to be uniformly ultimately bounded (UUB) by using Lyapunov theorem, and tracking error can converge to a small neighborhood of zero by choosing the design parameters appropriately. In the end, the simulation example of motor vessel "yukun" is employed to illustrate the effectiveness of the proposed algorithm.

## Keywords

*Ship Course; Discrete-Time Nonlinear System; RBF Neural Network*

## Introduction

In the past decades, control over ship motion has attracted an ever increasing interest; especially for the ship course control, since its properties have a great effect on the security and economy of ship navigation. Due to the characteristics of ship motion such as large inertia, time delay and nonlinear etc., the ship course control system has always been a uncertain nonlinear system being a complex control problem. In recent years, many control methods have been utilized in ship course control systems, for instance robust

control (Yang, 1999; Luo, 2009), VSC control (Yang, 1998), backstepping design (Wang, 2002) and adaptive control (Liu, 2012). These control schemes are in continuous-time domain.

However, along with the rapid expansion of maritime industry, the feature requirements of the ship maneuverability and control performance is more and more demanding. At the same time, ship is becoming large-scale, high speed and automation, therefore the traditional control scheme could't satisfy the reality requires any more. Thus, it is very important that the advanced controller for ship course should be studied. It is common knowledge that discrete-time systems rather than the continuous-time systems are the closest for describing a real plant. Following the development and modification of information processing technology and computer technology in the modern ship, the control of ship course with nonlinear system in discrete-time form has been a expected field (Song, 2007; Bai, 2009).

Since the universal approximation of the neural networks (NNs) has been proven (Park, 1991), the NNs have become an active research topic and obtained widespread attention in the control domain. Owing to the excellent function approximation ability, NNs have emerged as an attractive and powerful tool for stability analysis of complex nonlinear dynamic systems (Li, 2004; Ge, 1998; Lewis, 1999). NN control has been extensively studied for both continuous-time and discrete-time systems (Ge, 2002; Hovakimyan 2002; Jagannathan, 2006; Ge, 2003).

In comparison with continuous-time control design, NN control design of discrete-time systems is more difficult due to the lack of mathematical tools in discrete time. For instance, Lyapunov design for nonlinear discrete-time systems has become much more intractable than that for continuous-time systems

because the linearity property of the derivative of a lyapunov function in continuous time is not present in the difference of lyapunov function in discrete time (Yeh, 1995; Song, 1993). In recent year, there have been many control schemes to be proposed for discrete-time nonlinear systems (Yang, 2010; Ge, 2008; Zhang, 2005). For example, for a class of a single-input-single-output (SISO) affine NARMA system, direct adaptive NN control was proposed in (Chen, 1995), then based on the implicit function theorem, direct adaptive RBF NN control scheme was studied for a class of discrete-time single-input single-output non-affine nonlinear systems (Zhang, 2002). Furthermore, it was developed with multilayer neural network (MNN) for general nonlinear autoregressive moving average with exogenous inputs (NARMAX) systems in (Ge, 2004).

Motivated by the previous works, in this paper, a direct adaptive RBF NN control would be studied for a class of ship course with uncertain discrete-time nonlinear system. In order to avoid the problem of some system states that are unmeasurable, the original ship course with uncertain discrete-time nonlinear system is transformed to input-output model. According to the proved existing implicit desired feedback control (IDFC), the direct adaptive RBF NN control scheme has been proposed for ship course discrete-time uncertain nonlinear system. The RBF neural network are used as emulator of the Implicit Desired Feedback Control and approximate unknown function. It is proven that all the signals of the closed-loop system are uniformly ultimately bounded and the tracking errors converge to a bounded compact set. Finally, simulation example of the ship "yukun" illustrates the effectiveness of the proposed approach with desirable performance.

## Problem Formulation

### System Description

In general, the relationship between course  $\varphi$  and rudder angle  $\delta$  in the mathematical model of ship course control system can be described as follows (Jia, 1997):

$$\ddot{\varphi} + \frac{1}{T}H(\dot{\varphi}) = \frac{K}{T}\delta \quad (1)$$

where  $K$  is a constant for rotary of ship,  $T$  is a constant for trackability of ship.  $H(\dot{\varphi})$  is a nonlinear function on  $\dot{\varphi}$ , which can be indicated as follow:

$$H(\dot{\varphi}) = a_1\dot{\varphi} + a_2\dot{\varphi}^3 + a_3\dot{\varphi}^5 + \dots \quad (2)$$

where  $a_i, i = 1, 2, \dots$  are positive constants, which denotes nonlinear coefficients of the ship.

Based on formula (1) and (2), the following ship course with discrete-time nonlinear systems is obtained (Zhang, 2009):

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = -\frac{1}{T}(a_1x_2(k) + a_2x_2^3(k)) + \frac{K}{T}u(k) \\ y(k) = x_1(k) \end{cases} \quad (3)$$

where  $x_1, x_2 \in R$ ,  $y(k) \in R$  and  $u(k) \in R$  are the state variables, system output and input respectively.

According to literature (Liu, 2013), this discrete-time system (3) can be described as the following form:

$$y(k+1) = f(y(k), u(k)) \quad (4)$$

this model with relation of an input sequence  $\{u(k)\}$  to an output sequence  $\{y(k)\}$  by nonlinear difference equation, constitutes an extremely broad class, including many other classes of nonlinear discrete-time model as special cases.

The control objective is to design a direct RBF NN controller for system (3) such that the system output follows the desired reference signal  $y_d(k)$ . In the following, the following assumptions have been made based on the systems (3).

**Assumption 1** The nonlinear function  $f(\cdot)$  is unknown, continuous and differentiable.

**Assumption 2** The neural number of hidden neurons is  $l$ .

**Assumption 3** Assume that the partial derivative  $g_1 \geq \left| \frac{\partial f}{\partial u} \right| > \varepsilon > 0$ , where both  $\varepsilon$  and  $g_1$  are positive constants.

This assumption states that the partial derivative  $\frac{\partial f}{\partial u}$  is either positive or negative. From now onwards, without lose of generality, it is supposed that  $\frac{\partial f}{\partial u} > 0$ .

**Lemma 1** (Mean Value Theorem). If a function  $f(x)$  is continuous on the closed interval  $[a, b]$  and differentiable on the open interval  $(a, b)$ , then a point  $c$  exists in  $(a, b)$  such that:

$$f(b) = f(a) + (b - a) f'(c) \Big|_{c \in (a,b)} \quad (5)$$

### Radial Basis Function NN

In control engineering, RBF neural networks usually are used as a tool to model nonlinear functions because of their good capabilities in function approximation. The RBF can be considered as a two-layer network in which the hidden layer performs a fixed nonlinear transformation with no adjustable parameters, i.e., the input space is mapped into a new space. The output layer then combines the outputs in the latter space linearly. Therefore, they belong to a class of linearly parameterized networks. In this paper, the following RBF is used to approximate the continuous function  $s(z) : R^q \rightarrow R$ :

$$s_{nn}(z) = w^T h(z) \quad (6)$$

where the input vector  $z \in \Omega_z \subset R^q$ , where  $q$  is the neural network input dimension. Weight vector  $w = [w_1, w_2, \dots, w_l]^T \in R^l$ , the NN node number  $l > 1$ ; and  $h(z) = [h_1(z), \dots, h_l(z)]^T$ , with  $h_i(z)$  chosen as the commonly used Gaussian function, which is in the following form

$$h_i(z) = \exp \left[ \frac{-(z - \mu_i)^T (z - \mu_i)}{\eta_i^2} \right], i = 1, 2, \dots, l \quad (7)$$

where  $\mu_i = [\mu_{i1}, \mu_{i2}, \dots, \mu_{iq}]^T$  is the center of the receptive field and  $\eta_i$  is the width of the Gaussian function.

It has been proven that network (6) can approximate any continuous function over a compact set  $\Omega_z \subset R^q$  to arbitrary accuracy as

$$s(z) = w^{*T} h(z) + \varepsilon_z, \forall z \in \Omega_z \quad (8)$$

where  $w^*$  is ideal constant weights, and  $\varepsilon_z$  is the approximation error.

The ideal weight vector  $w^*$  is an "artificial" quantity required for analytical purposes.  $w^*$  is defined as the value of  $w$  that minimizes  $|\varepsilon_z|$  for all  $z \in \Omega_z$  in a compact region, i.e.

$$w^* := \arg \min_{w \in R^l} \left\{ \sup_{z \in \Omega_z} |s(z) - w^T h(z)| \right\}, z \in \Omega_z \quad (9)$$

### Controller Design and Stability Analysis

Assume that  $y_d(k+1)$  is the system's desired output

at time  $k+1$ . In the ideal case, there is no disturbance; and it can be shown that if the control input  $u^*(k)$  satisfies

$$f(y(k), u^*(k)) - y_d(k+1) = 0 \quad (10)$$

then the system's output tracking error will converge to zero.

**Definition 1** If a controller  $u^*(k)$  satisfying equation (10) exists, then the controller will drive the system output to the desired output, in addition, control input  $u^*(k)$  is called Implicit Desired Feedback Control (IDFC).

the tracking error is defined as

$$e(k) = y(k) - y_d(k) \quad (11)$$

and then the tracking error dynamic equation is given as follows:

$$e(k+1) = f(y(k), u(k)) - y_d(k+1) \quad (12)$$

For the system (4), a RBF neural network was designed to realize the controller directly [24]. The ideal control input is  $u^*$ , and

$$u^*(k) = u^*(z) = w^{*T} h(z) + \varepsilon_u(z) \quad (13)$$

on the compact set  $\Omega_z$ , the ideal neural network weights  $w^*$  and the approximation error are bounded by

$$\|w^*\| \leq w_m, |\varepsilon_u(z)| \leq \varepsilon_1 \quad (14)$$

where  $w_m$  and  $\varepsilon_1$  positive constants.

$\hat{w}(k)$  is defined as the actual neural network weight value, and the control law is designed by using RBF neural network directly

$$u(k) = \hat{w}^T(k) h(z) \quad (15)$$

where  $h(z)$  is Gaussian function output and  $z(k)$  is input of RBF.

By noticing (13), we have

$$\begin{aligned} u(k) - u^*(k) &= \hat{w}^T(k) h(z) - (w^{*T} h(z) + \varepsilon_u(z)) \\ &= \tilde{w}^T(k) h(z) - \varepsilon_u(z) \end{aligned} \quad (16)$$

where  $\tilde{w}(k) = \hat{w}(k) - w^*$  is the weight approximation error.

The weight update law was designed as follows [24]:

$$\hat{w}(k+1) = \hat{w}(k) - \gamma(h(z)e(k+1) + \sigma \hat{w}(k)) \quad (17)$$

where  $\gamma > 0$  is a positive number and  $\sigma > 0$ .

Subtracting  $w^*$  to both sides of (16), we have

$$\tilde{w}(k+1) = \tilde{w}(k) - \gamma(h(z)e(k+1) + \sigma\hat{w}(k)) \quad (18)$$

Using mean value theorem, let  $a = u^*(k)$ ,  $c = \varsigma$ ,  $b = u(k) = u^*(k) + \tilde{w}^T(k)h(z) - \varepsilon_u(z)$ . noticing Eqs. (10) and (16), then

$$\begin{aligned} f(y(k), u(k)) &= f(y(k), u^*(k) + \tilde{w}^T(k)h(z) - \varepsilon_u(z)) \\ &= y_d(k+1) + (\tilde{w}^T(k)h(z) - \varepsilon_u(z))f_u \end{aligned}$$

$$\text{where } f_u = \left. \frac{\partial f}{\partial u} \right|_{u=\varsigma}, \varsigma \in (u^*(k), u(k)).$$

then we get

$$\begin{aligned} e(k+1) &= f(y(k), u(k)) - y_d(k+1) \\ &= (\tilde{w}^T(k)h(z) - \varepsilon_u(z))f_u \end{aligned} \quad (19)$$

$$\text{and } \tilde{w}^T(k)h(z) = \frac{e(k+1)}{f_u} + \varepsilon_u(z) \quad (20)$$

The stability analysis of the closed system is given as follows:

Choose the Lyapunov function candidate

$$V(k) = \frac{1}{g_1}e^2(k) + \frac{1}{\gamma}\tilde{w}(k)^T\tilde{w}(k) \quad (21)$$

then we have

$$\begin{aligned} \Delta V &= V(k+1) - V(k) \\ &= \frac{1}{g_1}(e^2(k+1) - e^2(k)) - \frac{1}{\gamma}\tilde{w}(k)^T\tilde{w}(k) \\ &\quad + \frac{1}{\gamma}\tilde{w}(k+1)^T\tilde{w}(k+1) \\ &= \frac{1}{g_1}(e^2(k+1) - e^2(k)) - \frac{1}{\gamma}\tilde{w}(k)^T\tilde{w}(k) \\ &\quad + \frac{1}{\gamma}(\tilde{w}(k) - \gamma(h(z)e(k+1) + \sigma\hat{w}(k)))^T \\ &\quad \times (\tilde{w}(k) - \gamma(h(z)e(k+1) + \sigma\hat{w}(k))) \\ &= \frac{1}{g_1}(e^2(k+1) - e^2(k) - 2\tilde{w}(k)^T h(z)e(k+1) \\ &\quad - 2\sigma\tilde{w}(k)^T \hat{w}(k) + \gamma h^T(z)h(z)e^2(k+1) \\ &\quad + 2\gamma\sigma\hat{w}^T(k)h(z)e(k+1) + \gamma\sigma^2\hat{w}(k)^T \hat{w}(k)) \end{aligned}$$

Since

$$h^T(z)h(z) = \|h(z)\|^2 \leq l, i=1,2,\dots,l$$

$$2\sigma\tilde{w}(k)^T \hat{w}(k) = \sigma(\|\tilde{w}(k)\|^2 + \|\hat{w}(k)\|^2 - \|w^*\|^2)$$

$$\gamma h^T(z)h(z)e^2(k+1) \leq \gamma l e^2(k+1)$$

$$2\gamma\sigma\hat{w}^T(k)h(z)e(k+1) \leq \gamma\sigma l[\|\hat{w}(k)\|^2 + e^2(k+1)]$$

$$\gamma\sigma^2\hat{w}^T(k)\hat{w}(k) = \gamma\sigma^2\|\hat{w}(k)\|^2,$$

We obtain

$$\begin{aligned} \Delta V &\leq \frac{1}{g_1}(e^2(k+1) - e^2(k)) + \gamma\sigma^2\|\hat{w}(k)\|^2 \\ &\quad - 2\left(\frac{e(k+1)}{f_u} + \varepsilon_u(z)\right)e(k+1) \\ &\quad - \sigma(\|\tilde{w}(k)\|^2 + \|\hat{w}(k)\|^2 - \|w^*\|^2) \\ &\quad + \gamma l e^2(k+1) + \gamma\sigma l[\|\hat{w}(k)\|^2 + e^2(k+1)] \\ &\leq \left(\frac{1}{g_1} - \frac{2}{f_u} + \gamma(1+\sigma)l\right)e^2(k+1) - \frac{1}{g_1}e^2(k) \\ &\quad - 2\varepsilon_u(z)e(k+1) - \sigma\|\tilde{w}(k)\|^2 + \sigma\|w^*\|^2 \\ &\quad + \sigma(-3 + \gamma l + \gamma\sigma)\|\hat{w}(k)\|^2 \end{aligned}$$

Based on the assumption (13), from  $0 < \varepsilon < f_u \leq g_1$ , it can be deduced that

$$\frac{1}{g_1} - \frac{2}{f_u} \leq \frac{1}{g_1} - \frac{2}{g_1} = -\frac{1}{g_1} < 0,$$

$$-2\varepsilon_u(z)e(k+1) \leq k_0\varepsilon_1^2 + \frac{1}{k_0}e^2(k+1)$$

where  $k_0$  is a positive number. Thus, we have

$$\begin{aligned} \Delta V &\leq \left(-\frac{1}{g_1} + \gamma(1+\sigma)l + \frac{1}{k_0}\right)e^2(k+1) \\ &\quad + \sigma(\gamma l + \gamma\sigma - 1)\|\hat{w}(k)\|^2 - \frac{1}{g_1}e^2(k) \\ &\quad - \sigma\|\tilde{w}(k)\|^2 + \sigma w_m^2 + k_0\varepsilon_1^2 \\ &= -\left(\frac{1}{g_1} - (1+\sigma)l\gamma - \frac{1}{k_0}\right)e^2(k+1) \\ &\quad + \sigma((l+\sigma)\gamma - 3)\|\hat{w}(k)\|^2 - \sigma\|\tilde{w}(k)\|^2 \\ &\quad - (e^2(k) - \beta)/g_1 \end{aligned}$$

where  $\beta$  is a positive number as

$$\beta = g_1(\sigma w_m^2 + k_0\varepsilon_1^2) \quad (22)$$

Choosing the positive constants as  $k_0$ ,  $\lambda$  and  $\sigma$ ,

these constants must be satisfied the following inequalities:

$$\frac{1}{g_1} - \frac{1}{k_0} \geq 0, \quad \frac{1}{g_1} - (1+\sigma)l\gamma - \frac{1}{k_0} \geq 0, \quad (l+\sigma)\gamma - 3 \leq 0,$$

that is,

$$0 < g_1 \leq k_0 \quad (23)$$

$$0 < (1+\sigma)l\gamma \leq \frac{1}{g_1} - \frac{1}{k_0} \quad (24)$$

$$0 < (1+\sigma)\gamma \leq 3 \quad (25)$$

then we have  $\Delta V \leq 0$  once  $e^2(k) \geq \beta$ . This states that for all  $k \geq 0$ ,  $V(k)$  is bounded because

$$V(k) = V(0) + \sum_{j=0}^k \Delta V(i) < \infty$$

Define compact set  $\Omega_e = \{e | e^2 \leq \beta\}$ , then it can be seen that the tracking error  $e(k)$  will converge to  $\Omega_e$  if  $e(k)$  is out of compact  $\Omega_e$ .

### Simulation Study

To illustrate the effectiveness of the proposed control scheme, the motor vessel "yukun" of Dalian Maritime University is used to implement the simulation.

Consider the ship course discrete-time nonlinear system as:

$$\begin{cases} x_1(k+1) = x_2(k) \\ x_2(k+1) = -\frac{1}{T}(a_1 x_2(k) + a_2 x_2^3(k)) + \frac{K}{T} u(k) \\ y(k) = x_1(k) \end{cases}$$

the ship's length  $L$  is 116 m, breadth  $B$  is 18 m, draft  $d$  is 5.4 m, and the speed is 16.7 kn. Therefore, the mathematical model parameters of ship's nonlinear motion is  $K = 0.5$ ,  $T = 64$ ,  $a_1 = 0.5$ ,  $a_2 = 30$ . The tracking objective is to make the output  $y$  follow a desired reference signal that is  $y_m(k) = 30 \sin(k\pi/180)$ .

Since  $g_1 \geq \partial f / \partial u = 1/64$ , thus we can set  $g_1 = 0.02$ , and from (23), we can choose  $k_0 = 0.05$ . For RBF neural network, the NN node number  $l = 21$ . The initial value of the plant is  $x(0) = [0, 0]^T$ . To satisfy (24) and (25), we can choose  $\gamma = 0.11$ ,  $\sigma = 0.9$ .

The simulation results are shown in Figs. 1, 2 and 3. Fig. 1 shows the tracking performance for output  $y$  relative to the reference signal  $y_d(k)$  obtained. Fig. 2 and Fig. 3 illustrate the control input signal of the closed-loop system and the norm of RBF neural network weight respectively.

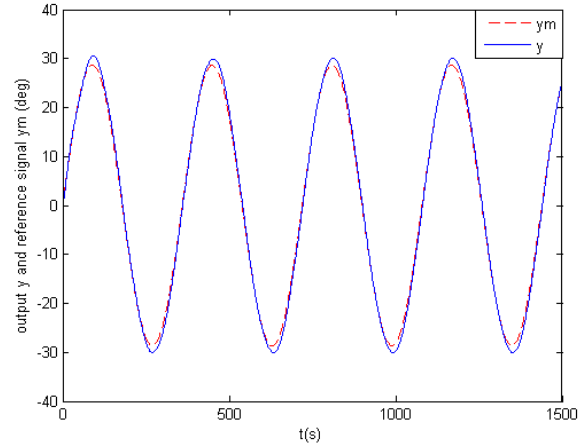


FIG. 1 POSITION TRACKING

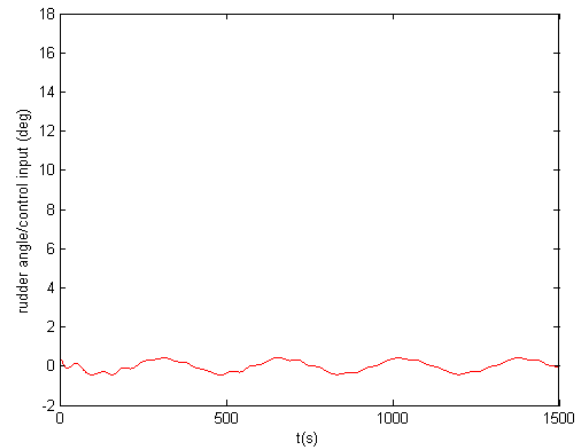


FIG. 2 CONTROL INPUT

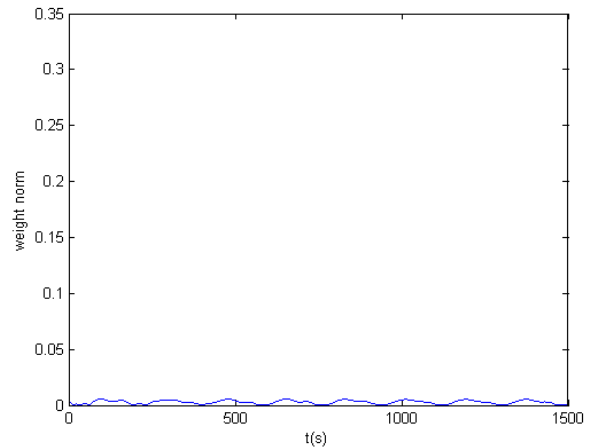


FIG. 3 WEIGHT NORM

## Conclusion

In this paper, a class of direct adaptive RBF neural network controller is presented for the ship course with uncertain discrete-time nonlinear system. Motivated by the previous works, the ship course with discrete-time nonlinear system is transformed to input-output model, in which the existence and uniqueness of implicit desired feedback control have been proven by Implicit function theorem. Then based on the input-output model, RBF neural networks are used as emulator of the Implicit Desired Feedback Control. All the signals of the closed-loop system are guaranteed to be uniformly ultimately bounded, and the tracking error can be made arbitrarily small by choosing the parameters in the control law. Simulation example of the real ship "yukun" has been carried out to demonstrate the performance and the effectiveness of the proposed algorithm.

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